2011-2012
IB MYP YEAR 4

SUMMATIVE ASSESSMENT

Dathe

Year 9 Mathematics (Extended)

Name: Cindy Cheng [9 Peace]

Date of task: **8th June, 2012** Time allowed: **1.5 hours (11:40 -13:10)**

Teacher: **Ms Li / Mr Millard / Mr So**

Student's **Performance** in Different **Criteria**			
A	5	C	5

Instructions

- Read the instructions for all questions carefully.
- All work must be hand written.
- All work, steps and proper units must be shown.
- A non-electronic dictionary is allowed.
- Use of calculator is allowed.

Advice:

- Read the criteria descriptors and task-specific rubrics carefully before you start your work. This will give you a clear understanding of what is required and what a high quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest levels in this task.

- This assessment task will be assessed on Criterion **A & C**.
  - For Criteria **A**, the questions are all assigned with levels;
  - Criterion **C** will be assessed as an overall impression on the presentation of work in this assessment.

ASSESSMENT CRITERIA

Criterion A: KNOWLEDGE AND UNDERSTANDING

Achievement level	Task Specific Rubric	IBO Published Descriptor
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.
1–2 **Simple**	The student can solve **some** **simple** problems.	The student **generally** makes appropriate deductions when solving **simple** problems in **familiar** contexts.
3–4 **Complex**	The student can solve **most** **simple** and **some** **more complex** problems.	The student **generally** makes appropriate deductions when solving **more complex** problems in **familiar** contexts.
5–6 **Challenging**	The student can solve **some** **challenging** problem along with **all** different types of problems.	The student **generally** makes **appropriate** deductions when solving **challenging** problems in a **variety** of **familiar** contexts.
7–8 **Unfamiliar**	The student can solve **most** challenging and **most** **unfamiliar** problems along with **all** different types of problems.	The student **consistently** makes **appropriate** deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar** situations.

Criterion C: COMMUNICATION IN MATHEMATICS

Achievement level	Task Specific Rubric	IBO Published Descriptor
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.
1–2	The student should be able to explain **some problems** step by step. The lines of reasoning are **difficult to follow**.	The student shows **basic** use of mathematical language **and/or** forms of mathematical representation. The lines of reasoning are **difficult to follow**.
3–4	The student should be able to explain **most problems** step by step. The lines of reasoning are **clear** though **not always** **logical** or **complete**.	The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **clear** though not always **logical** or **complete**. The student moves between different forms of representation **with some success.**
5–6	The student should be able to explain **most problems** step by step. The lines of reasoning are **concise**, **logical** and **complete**. The student use **correct unit** in the questions.	The student shows **good** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **concise**, **logical** and **complete**. The student moves **effectively** between different forms of representation.

A. Simple problems

Suggested time allocation for Question **1 to 5** is **15 minutes**.

1. Given the points A $(-1, 2)$ and $B = (2, k)$, find the value(s) of k such that the **length of line AB is 5 units**.

$5 = \sqrt{(-1+2)^2 + (2+k)^2}$

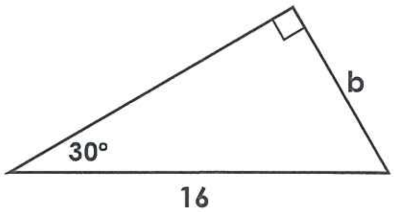
$5 = 1 + 4 + k^2$

$5 = 5 + k^2$

$-k^2 = 5 - 5$

$k = 0$

2. In the figure on the right, find the value of **b** **without using calculator**.



$\sin 30° = \frac{b}{16}$

$\frac{1}{2} = \frac{b}{16}$

$2b = 16$

$b = 8$

3. Given that the equation of the line **L1** is $y - 2x = 4$, which of the following line(s) is/are **parallel to L1**? Which of the following line(s) has/have **negative y-intercepts**?

$y = 2x + 4$

$-3y = \frac{+6x}{+3}$

L2: $y = -2x + 4$ **L3:** $2y - 4x - 5 = 0$ **L4:** $-3y = 2x + 4$ **L5:** $6x - 9 = 3y$

$y = \frac{4x+5}{2}$ $y = \frac{2x}{-3} + \frac{4}{-3}$ $y = 2x - 3$

Explain your answers by showing your calculations.

$L3: y = \frac{4x}{2} + \frac{5}{2}$

$m_{L3} = \frac{4}{2}$

$= 2$

$m_{L1} = 2$

$\therefore L1 // L3$

$L5: -3y = -6x + 9$

$y = \frac{-6x}{-3} + \frac{9}{-3}$

$y = 2x - 3$

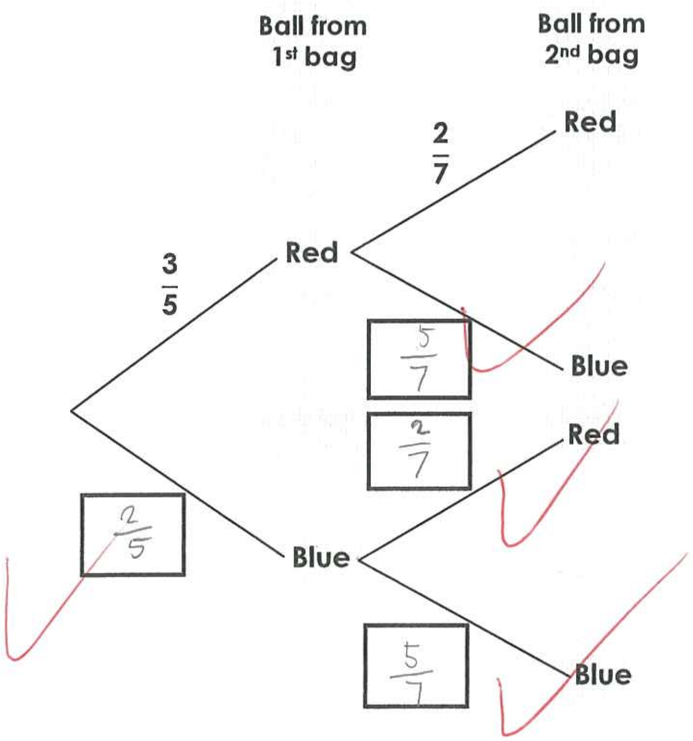
$\therefore$ L5 have negative y-intercept.

?

?

4. Loren has two bags. The **first** bag contains **3 red** balls and **2 blue** balls. The **second** bag contains **2 red** balls and **5 blue** balls. Loren takes **1 ball** at random from **each bag**.

(a) Complete the probability **tree diagram** by entering the **correct answers into the boxes.**



(b) Find the probability that Loren takes **two red balls**.

$P(r) = \frac{5}{12}$

5. Evaluate the following **without using calculator**.

$$\sin^2 23° + \cos^2 23° - \frac{\sin 45°}{\cos 45°}$$

$\sin^2 23° + \cos^2 23° - \frac{\sin 45°}{\cos 45°}$

$= 1 - \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

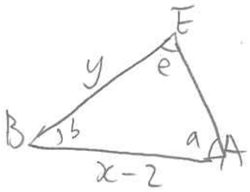
$= 1 - 1$

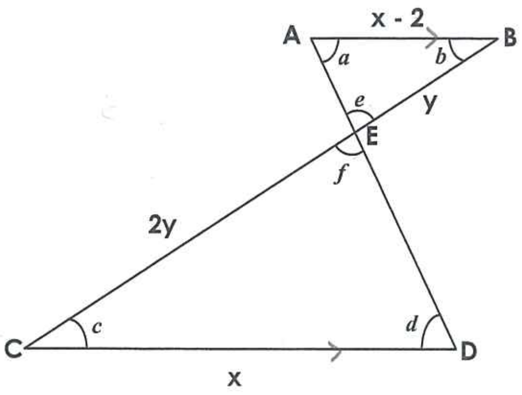
$= 0$

B. MORE COMPLEX PROBLEMS

Suggested time allocation for Question **6 to 9** is **25 minutes**.

6. In the figure below, the line AB is parallel to the line CD and some dimensions are shown in terms of x or y.





(a) Show that Δ **ABE** and Δ**DCE** are **similar**. State the reason(s) if necessary.

$\angle a = \angle d$ (alt ∠s, AB//CD)

$\angle b = \angle c$ (alt ∠s, AB//CD)

$\angle e = \angle f$ (vert. opp. ∠s)

∴ $\triangle ABE \sim \triangle DCE$ (equiangular)

(b) Find the value of **x**.

$$\frac{x-2}{x} = \frac{y}{2y}$$

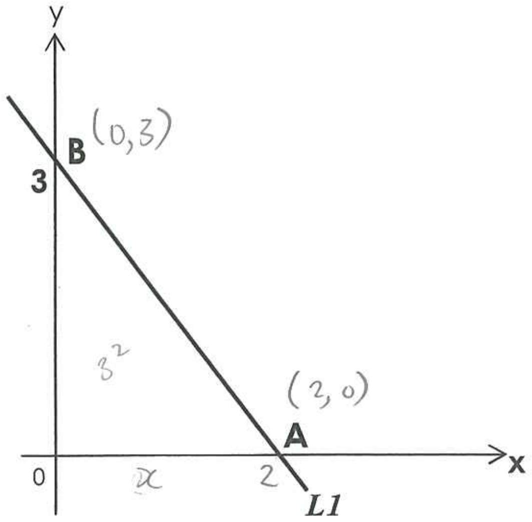
$$\frac{x-2}{x} = \frac{1}{2}$$

$$x = 2x - 4$$

$$-x = -4$$

$$x = 4$$

7. In the graph on the right, a line $L1$ cuts the x-axis and y-axis at point **A** and **B** respectively. The y-intercept is 3.



(a) If the area of the triangle AOB is 3 square units, find the **equation** of $L1$. Express your answer in **slope-intercept form**.

$\frac{3x}{2} = 3$

$3x = 6$

$x = 2$

$m_{AB} = \frac{3-0}{0-2}$

$= \frac{3}{-2}$

$L_1: y = \frac{3}{-2}x + 3$

(b) If a line $L2$ is **perpendicular** to $L1$ and two lines intersect at point **D(4,-3)**, find the equation of $L2$. Express your answer in **general form**.

$m_{L_1} \times m_{L_2} = -1$

$\frac{3}{-2} \times m_{L_2} = -1$

$m_{L_2} = \frac{2}{3}$

$y - (-3) = \frac{2}{3}(x-4)$

$y + 3 = \frac{2}{3}x - \frac{8}{3}$

$y = \frac{2}{3}x - \frac{8}{3} - 3$

$y = \frac{2}{3}x - \frac{17}{3}$

$\therefore$ general form: $-\frac{2}{3}x + y + \frac{17}{3} = 0$

8. In a certain dice game, the player throws **two** typical unbiased **six-faces dice** and receives **$5** if the sum is **7 or 11**, otherwise he or she **pays $2**.

(a) Calculate the probability of obtaining the **sum of 7 or 11** when you **throw the two dice once**.

$P(s) = \frac{8}{36}$

1+1 2+1 3+1 4+1
1+2 2+2 3+2 4+2
1+3 2+3 3+3 4+3
1+4 2+4 3+4 4+4
1+5 2+5 3+5 4+5
1+6 2+6 3+6 4+6

5+1 6+1
5+2 6+2
5+3 6+3
5+4 6+4
5+5 6+5
5+6 6+6

(b) If you play the game **18 times**, calculate the **amount of money** you **expect to gain or lose**.

gain :

$\frac{8}{36} \times 18$

$= \frac{144}{648}$

144×5

$= \$720$

lose :

$\frac{648-144}{648}$

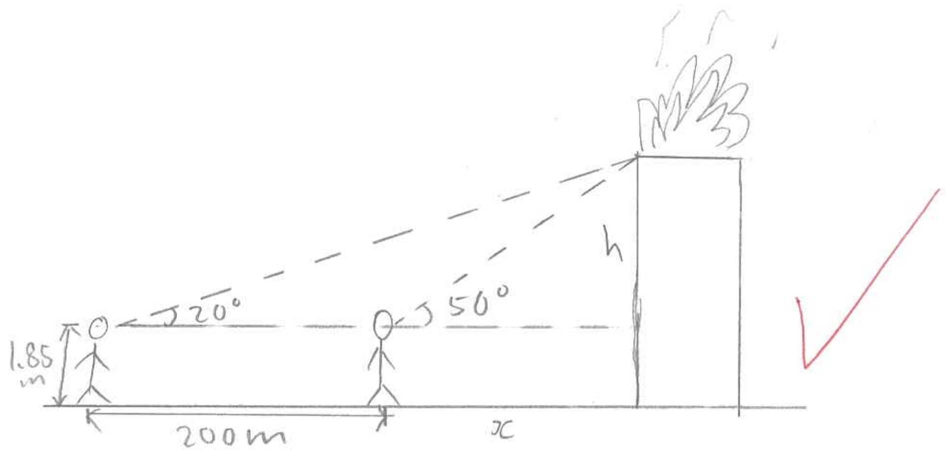
$= \frac{504}{648}$

504×2

$= \$1008$

9. Mr Bolivar, a volunteer fireman who is 1.85 m tall, is running towards a burning building where there is a fire on the roof. Initially, his angle of elevation to the roof is 20°. He runs for 200 m and now his angle of elevation is 50°. Assume that <u>the ground is horizontal and the building is vertical</u>.

(a) Sketch a **diagram** to represent the information above.



(b) How tall is the building? Correct your answer to the **nearest meter**.

$\tan 20° = \frac{h}{200+x}$

$\cancel{200+}x = \frac{h}{\tan 20°} - 200$

$\tan 50° = \frac{h}{x}$

$h = x \times \tan 50°$

$x = \frac{h}{\tan 50°}$

$\frac{h}{\tan 20°} - 200 - \frac{h}{\tan 50°} = 200$

$\frac{h}{\tan 20°} - \frac{h}{\tan 50°} = 400$

$\frac{h}{\tan 20°} = \frac{h}{\tan 50°} + 400$

$h \tan 20° = h \tan 50° + 400$

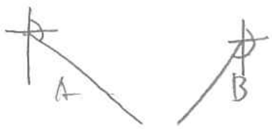
$\frac{h}{h} = \frac{\cancel{\tan 50°}}{\cancel{\tan 20°}} + 400$

$h = 2.5 + 400$

$h = 402.5$

$\therefore 402.5 + 1.85$

$= 404 \text{ m}$ //



C. CHALLENGING PROBLEM

Suggested time allocation for Question **10 and 11** is **30 minutes**.

10. Ship **A** leaves the harbor H on a bearing **150°** with a speed of **40** km/hr. At the same time, Ship **B** leaves harbor H on a bearing **210°** with a speed of **40** km/hr.

(a) **After 12 minutes,** how far did ship A and ship B travel?

$s = \frac{d}{t}$

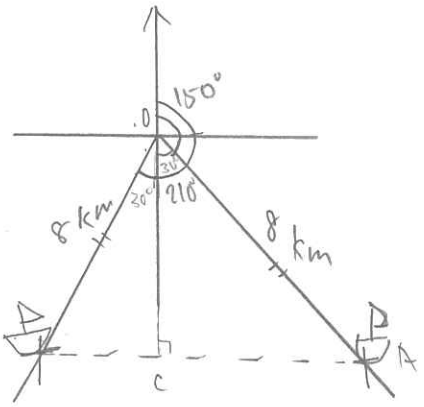
~~$d = 12 \times 40$~~

~~$d = 480$~~

0.67×12

$= 8$ km

(b) Sketch a **diagram** to represent the information above **12 minutes after the two ships left the harbor.**



(c) Find the **true bearing from Ship A to Ship B** 12 minutes after they left the harbor.

090°

(d) Find the **distance between the two ships** 12 minutes after they left the harbor. Give your answer to the **nearest meter.**

$180° - 150° = 30°$

$210° - 180° = 30°$

$\sin 30° = \frac{CA}{8}$

$\frac{1}{2} = \frac{CA}{8}$

$2CA = 8$

$CA = 4$

$\because \angle BOC = \angle COA$

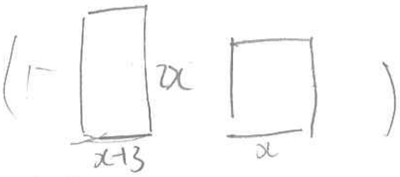
$BO = AO$

$\therefore BC = CA$

$\therefore BC + CA = BA$

$4 + 4 = BA$

$BA = 8$



11. The properties of a rectangle and a square are given below:

- The length of the rectangle is 3 cm longer than the side of the square.
- The width of the rectangle is double the length of the side of the square.

If the **sum of their areas** is **24 cm²**, find the **dimensions** (that is, its length and width) of the rectangle.

$(2x)(x+3)+x^2=24$

$(2x^2+6x)+x^2=24$

$6x+3x^2=24$

$3x=\frac{24}{6x}$

$3x^2=4x$

$\frac{3x^2}{x}=4$

$3x=4$

$x=\frac{4}{3}$

length: $\frac{4}{3}+3$

$=\frac{13}{3}$ cm

width: $\frac{4}{3}\times 2$

$=\frac{8}{3}$ cm

$(y-3\times\frac{x}{2})+xy=24$

$\frac{(y-3)x}{2}+xy=24$

$\frac{xy-3x}{2}+xy=24$

$xy-3x+xy=48$

$2xy-3x=48$

D. Unfamiliar problems (Suggested time allocation for Question **12 and 13** is **30 minutes**.)

12. At noon, Tom and Pete both park at the same starting point. Tom starts to ride his bike at 8 miles/hr. Two hours later, Pete starts after Tom on a bicycle at 12 miles/hr.

(a) How far will Tom have ridden before he is **overtaken by Pete**?

$d = 8 \times 2$

$= 16$ miles

(b) At what time will Tom and Pete be **8 miles** apart?

$d = 12 \times 2$

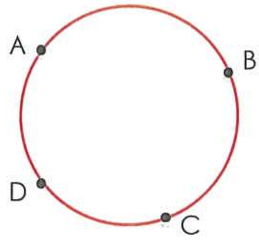
$= 24$

$24 - 16 = 8$

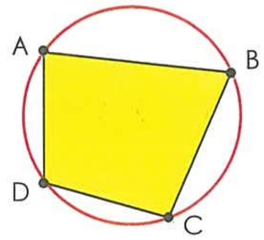
2 hours.

13. Please read the following information and then do the proof on next page.

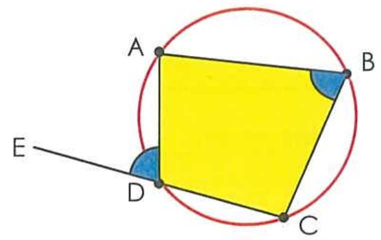
Points lie on the **same circle**, as the diagram below, are said to be **concyclic**. For example, A, B, C and D are **concyclic points.**



If the vertices of a **quadrilateral** lie on a **circle**, as the diagram below, then the quadrilateral is said to be **cyclic.** For example, ABCD is a **cyclic quadrilateral** since the vertices A, B, C and D lie on the circle.

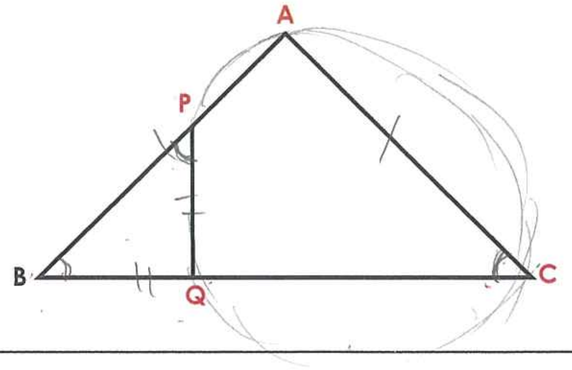


If the side CD is produced (i.e. extended) to E, as the diagram below, then $\angle$**ADE** is called the **exterior angle of the cyclic quadrilateral** ABCD, and $\angle$**ABC** is said to be the **interior opposite angle.**



Theorem: **If $\angle ADE = \angle ABC$, then A, B, C and D are concyclic. (ext.$\angle$, int. opp.$\angle$)**

In the figure below, **ΔABC** and **ΔBPQ** are **isosceles** triangles such that **AB = AC** and **BQ = PQ**.
Using the provided information about the concyclic points and cyclic quadrilateral, **prove** that **A, P, Q and C are concyclic**.



$\angle C = \angle B$ (base $\angle$s isos Δ)

$\angle BPQ = \angle B$ (")

$\therefore \angle C = \angle B = \angle P$

$\therefore \angle C = \angle BPQ$ (ext. $\angle$, int. opp. $\angle$)

$\therefore$ A, P, Q, C are concyclic.

End of Assessment